Wavelet Harmonic Balance

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***Abstract*—This letter introduces a new approach to steady state analysis of nonlinear microwave circuits under periodic excitation. The new method is similar to the well known technique of Harmonic Balance, but uses wavelets as basis functions instead of Fourier series. Use of wavelets allows significant increase in sparsity of the equation matrices and consequently decrease in CPU cost and storage requirements, while retaining accuracy and convergence of the traditional approach. The new method scales linearly with the size of the problem and is well suited for large scale simulations.**

***Index Terms*—Harmonic balance, nonlinear circuits, steady state analysis, wavelets.**

I. INTRODUCTION

STEADY state analysis of nonlinear circuits represents one of the most computationally challenging problems in microwave circuit design. The traditional approach of Harmonic Balance [1] assumes obtaining the solution x of the nonlinear MNA equation [2]

$$C\dot{x} + Gx + f(x) + u = 0 \qquad (1)$$

that satisfies periodical boundary conditions

$$x(t+\tau) = x(t), \quad u(t+\tau) = u(t). \qquad (2)$$

Solution is obtained by expanding (1) in Fourier basis that is naturally periodic and thus enforces the boundary conditions. This results in a system of nonlinear algebraic equations in frequency domain

$$\Phi(X) = (\hat{C}D + \hat{G})X + F(X) + U = 0,$$
$$X = Tx,\ x = \tilde{T}X,\ U = Tu \qquad (3)$$

where T and $\tilde{T}$ are matrices associated with the forward and inverse Fourier transform. This system is solved by Newton's iterations. Jacobian for (3) can be written in the following form:

$$J(X) = \frac{\partial \Phi}{\partial X} = \hat{C}D + \hat{G} + T\left[\frac{\partial f_k}{\partial x_l}\right]\tilde{T}. \qquad (4)$$

Because Fourier basis functions (sines and cosines) have full support on an interval, matrices T and $\tilde{T}$ are essentially dense which results in appearance of structurally dense blocks in the Jacobian, non withstanding the fact that in HB simulators Fourier Transform is computed implicitly, using FFT algorithm. These dense blocks are associated with nonlinear elements in the circuit and their size is proportional to the number of harmonics to be computed. The over-all fairly high density of the Jacobian results in high computational expense (CPU time and memory requirements) incurred in the Harmonic Balance simulators, particularly for highly nonlinear and multitone circuits.

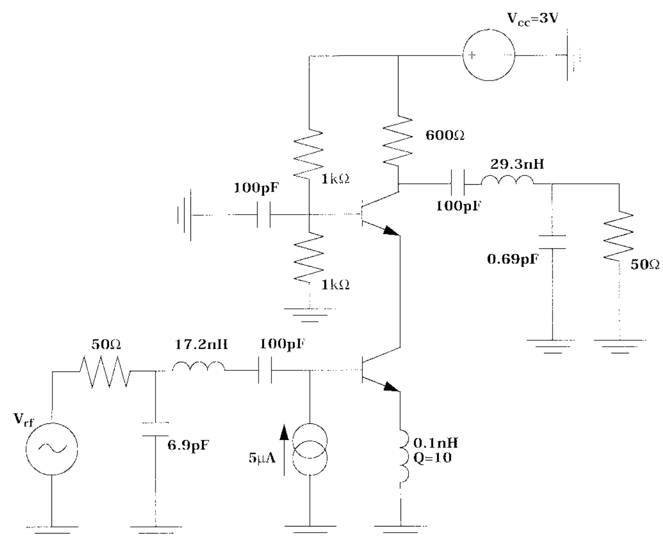

Fig. 1. Cascode amplifier circuit.

Different techniques have been previously proposed to improve Harmonic Balance (e.g., [3]–[6]). However, to the best of our knowledge, little, if any, research have been done in feasibility and advantages of expansion in bases other than Fourier series. Possibility of a wavelet formulation for nonlinear steady-state analysis was mentioned in [7], but the matter was not pursued any further.

We propose a completely new method, which we call Wavelet Harmonic Balance, and in which we use wavelets [8] as the expansion basis.

II. WAVELET FORMULATION

Full details of the wavelet formulation are undoubtedly beyond the limits of this letter and should be a subject of a separate paper. Here we can only briefly outline the major points of it.

Because of the general form matrix formulation, (3) and (4) still hold, with minor differences arising from the construction of the matrices in (3). Matrix D is the projection of the derivative operator onto subspace spanned by the expansion basis. In traditional Harmonic Balance, this matrix is diagonal in real Schur form. With wavelets, this matrix contains connection coefficients which define projection of the derivative operator onto wavelet space in nonstandard operator form [9]

$$D = TR\tilde{T} \qquad (5)$$

$$r_i = \left\langle \varphi(t-i), \frac{\partial}{\partial t}\varphi(t) \right\rangle \qquad (6)$$

where $\varphi(t)$ is the scaling function associated with given wavelet, matrix R has elements r_i on it's i-th diagonal and matrices T

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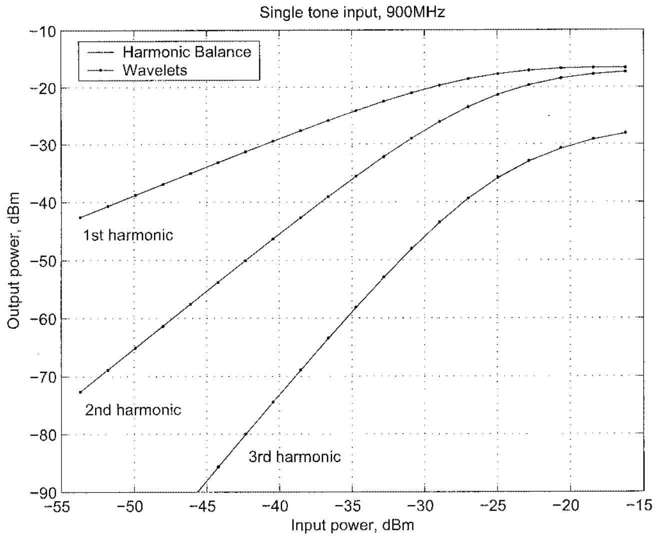

Fig. 2. Single tone input simulation results for the cascode amplifier in Fig. 1.

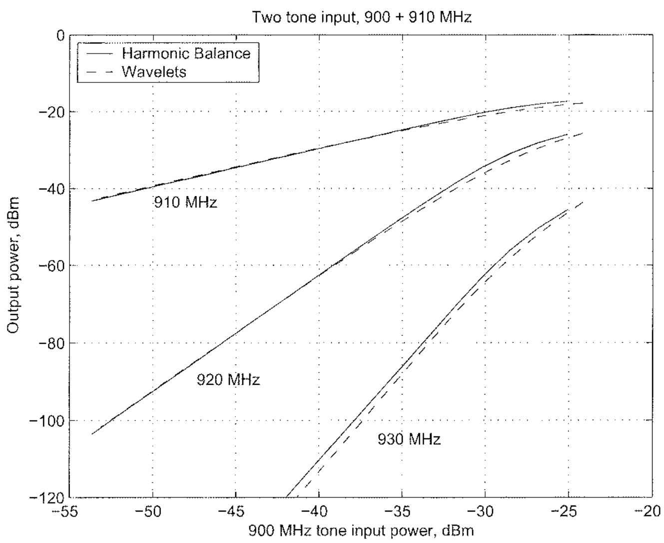

Fig. 3. Two tone input simulation results for the cascode amplifier in Fig. 1.

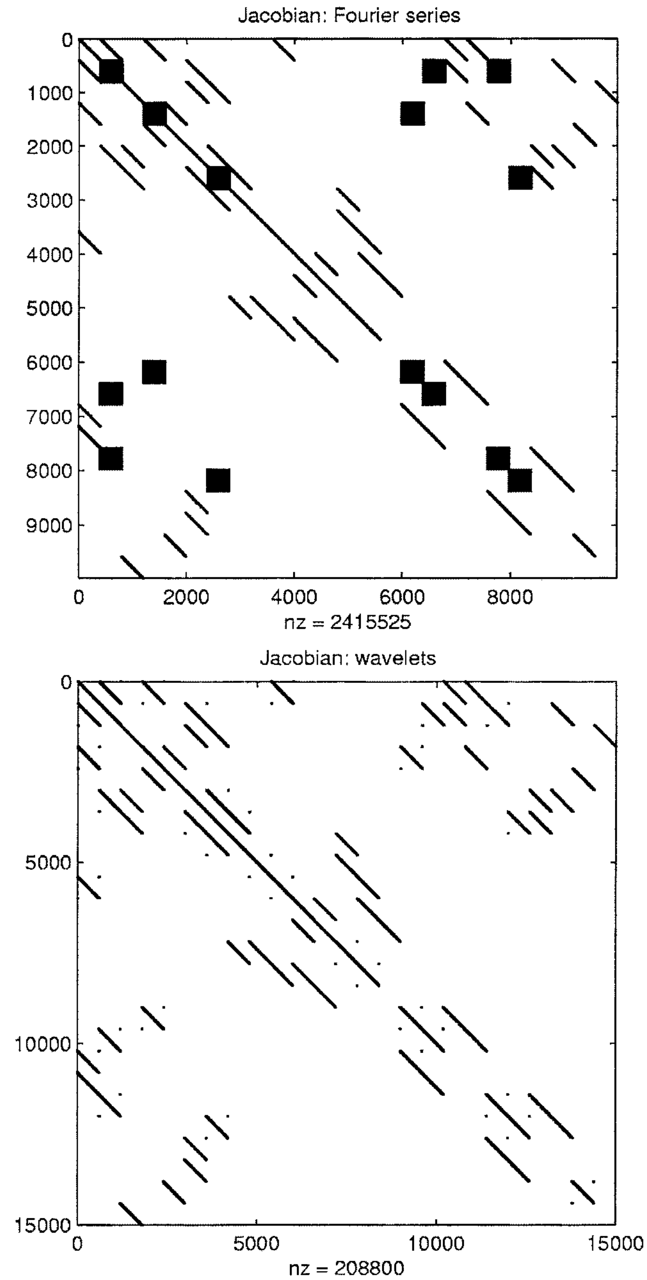

Fig. 4. Jacobian sparsity patterns.

and $\tilde{T}$ are associated with the forward and inverse wavelet transform. For each type of wavelets, connection coefficients are obtained as a solution of linear system of equations [10]

$$r_m = 2\left[r_{2m} + \frac{1}{2}\sum_{k=1}^{\frac{M}{2}} a_{2k-1}(r_{2m-2k+1} + r_{2m+2k-1})\right] \quad (7)$$

where a_i are autocorrelation coefficients of the low pass Quadrature Mirror Filters (QMF) associated with chosen (bi-)orthogonal wavelet basis

$$a_i = 2\sum_{m=0}^{M-i-1} \tilde{h}_m h_{m+i}, \quad i = \overline{1, \ldots, M-1}. \quad (8)$$

Because we can choose wavelets that have local support, the QMFs are essentially finite response filters and connection coefficients in (6) are nonzero only for small values of i. Transform matrices T and $\tilde{T}$ are constructed from QMF coefficients and thus are also sparse and bandlimited [11]. This means that in wavelet basis matrix D is sparse and bandlimited. Periodic boundary conditions are enforced by using periodized wavelets [12] which preserve bandwidth of the matrices D, T and $\tilde{T}$ and, consequently, that of the Jacobian (4).

III. Numerical Results

The proposed Wavelet Harmonic Balance method was used to simulate cascode amplifier circuit in Fig. 1. Second order orthogonal Daubechies wavelets were used for the wavelet formulation. Results of the single tone 900 Mhz input power sweep simulations are shown in Fig. 2. As one can see, results of the simulation using wavelet formulation are in excellent agreement with the traditional Harmonic Balance. Size of the Jacobian in this example was 775 × 775 with 16 470 nonzero entries for Fourier series 800 × 800 and with 10 827 nonzero entries for wavelets. In this example both methods exhibited essentially the same convergence and computational cost.

Second example involves the same circuit with two tone input signals of the same power, with frequencies of 900 and 910 MHz. Purpose of this experiment is to demonstrate speed and accuracy of the proposed method on computations of the third order in-band intermodulation products at 920 and 930 MHz. Simulation results are shown in Fig. 3 and are in very good correspondence with each other. Size of the Jacobian in this example was 9975 × 9975 with 2 415 525 nonzero entries (27.7 MBytes) for Fourier series and 15 000 × 15 000 with 203 397 nonzero entries (2.49 MBytes) for wavelets. Average time for one LU decomposition (on a 900 MHz SUN Blade-1000 workstation) of the Jacobian after symmetric approximate minimum degree preordering was 105 seconds for Fourier series versus 11 seconds for wavelets. Both methods also exhibited essentially the same convergence. Jacobian sparsity patterns for both cases are shown in Fig. 4.

IV. Concluding Remarks

We have researched the possibility of using wavelets as basis functions for expansion in Harmonic Balance-like method of simulation steady state response of nonlinear circuits under periodic excitations. This research resulted in the introduction of completely new technique for such simulations. We call this technique Wavelet Harmonic Balance. The new technique is as accurate as traditional Harmonic Balance, but has a potential of being much faster, particularly for multitone simulations.

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